

age-characteristics of the drainage-basin are not modified and the recession-curve in the range of discharge below the outlet top is of Type A. The value of M in this range is approximately 1.2. However, when the level of the pool is above the top of the outlet conduits, the value of M is reduced to 0.35, making the recession-hydrographs of Type E and concave downwards. Hence, when the reservoir drains, the recession-limb of the hydrograph is concave downwards until the level of the top of the fixed-outlet conduits is reached, when the recession-hydrograph becomes concave upwards. This example is given in preference to a natural river-basin because the discharge-channel-storage relationship derived as previously explained in this paper and whose characteristics have just been discussed, was checked by addition of the known volumes of water in the reservoir at a given discharge at the gaging station and volumes of channel-storage above a gaging station on the Stillwater River above the Reservoir at Pleasant Hill, Ohio, (drainage-area 502 square miles). The recession-curve at the latter place is of Type A, the value of M being approximately 1.1.

The upper end of the recession-curve shown on Figure 1, to which Harrold refers, is of Type E. It is not considered essential to the purpose of this paper to support the accuracy of the particular curve shown on Figure 1; subsequent data may support it or not. However, its accuracy cannot be challenged without adequate study of the hydraulic characteristics of the tributary basin. The fact that the value of M for high discharge-values is less than 0.5 is not in itself sufficient evidence to discredit it. Mr. Harrold's remarks concerning Figure 3 are well taken. Volumes of channel-storage in a given problem should be reckoned in second-foot-days and not in terms of unit channel-storage as shown on Figure 3. This figure is presented to demonstrate variations in unit channel-storage and to suggest a method for adjustment of discharge-channel-storage graphs.

SYNTHETIC UNIT-GRAPHS

Franklin F. Snyder

Synopsis--This paper presents a method, mostly empirical, of deriving synthetic unit-graphs. It is of assistance in the study and analysis of runoff-characteristics of drainage-areas of from 10 to 10,000 square miles for which stream-flow records may or may not be available. No attempt has been made to eliminate the use or need of judgment and experience in such studies.

The "lag" or time from center of mass of rainfall to peak of runoff is the principal drainage-basin characteristic used in deriving the synthetic unit-graphs. An approximate method of determining the "lag" is given for use on areas with no stage-records available. The peak-rate of the unit-graph is expressed as a function of the "lag".

A distribution-graph is also determined by means of the lag. Knowing the peak-rate and the distribution-graph for the area in question, a unit-graph can then be constructed. An example of this procedure is given for the French Broad River Basin above Dandridge, Tennessee.

The additional use of the lag-characteristic in flood-forecasting is described and in conclusion, the limitations of the method are discussed.

Introduction--It is assumed that the reader is familiar with the basic theory and practical application of unit-graphs. If not, reference is made to them [see 1, 2, and 3 of "References" at end of paper]. As used by the author, a unit-graph is the discharge-graph of one inch of surface-runoff from a given area for a typical or specified type of storm of some unit of duration. The basic assumptions of the unit-graph method are that for such storms the time-duration of discharge of surface-runoff is a constant for any basin and the ordinates are proportional to the total amount of surface-runoff and distributed according to the unit-graph shape. A distribution-graph expresses the unit-graph by giving the discharge-volume of successive equal time-periods as a percentage of the total volume.

The procedure herein described for deriving unit-graphs was developed mainly for studying and forecasting stream-flow on areas with no immediately available records of runoff. However, it has been of material assistance in the study of areas for which discharge or gage-height records are available.

True unit-graphs are not directly obtainable from published mean daily discharges, although distribution-graphs based on 24-hour periods can be worked up and serve as an aid in outlining the unit-graph.

Mean daily discharges may indicate a single smooth runoff-graph when actually two closely spaced rises may have occurred. Actual drainage-basin characteristics and irregularities may also be concealed. In addition, the fortuitous synchronization of runoff with the calendar day will affect the results. Some of these difficulties are reduced if recording rainfall-records are available but, until recently, this has not generally been the case.

Even with continuous records of river-stages available, and this is now the case at the majority of river-measurement stations, the investigator who is trying to derive unit-graphs for the first time is often discouraged. The usual procedure is to obtain unit-graphs for all stream-rises that are fairly well isolated and seemingly the result of storms of about the same duration. When these unit-graphs are compared, the variations may often seem too great for substantiation of the unit-graph method.

These differences are mainly explainable by variations in the lengths of the surface-run-off-producing storms, unequal distribution of rainfall, differences in storm-paths, and runoff from snow or ice if the area is so affected.

None of the conflicting results may be safely disregarded until the reasons for the discrepancies are determined. Likewise, the unit-graph desired may be one for a special or typical type of storm rather than one of uniform intensity and distribution of rainfall, especially if the basin is one on which the latter very seldom occurs.

The selection of a unit of duration is the next step. In studying small areas this may need to be as short as six or even one hour. The problem then is to develop a unit-graph for use with data of rainfall for one or six hours from the available unit-graphs based on storms of from probably six to 36 hours in duration. This can be done by trial and error or by systematic break-down of a unit-graph of known storm-duration into a unit-graph for use with rainfall-data of half the original duration. With care this process can be repeated until the desired graph is obtained. An idea of how to make this break-down can be obtained by graphically combining two of the smaller duration unit-graphs (for example, say 12 hours) staggered 12 hours and getting the resultant 24-hour unit-graph by adding the ordinates and dividing by two. One can then see how to carry out the reverse process.

The procedure for deriving synthetic unit-graphs presented in this paper can be used to advantage in obtaining a trial unit-graph. Regardless of how they are obtained, all unit-graphs should be tested by application to actual storms.

Determination of the "lag"-characteristic of any drainage-area--The term "lag" as introduced by W. W. Horner and F. L. Flynt in their study of rainfall and runoff from urban areas [4] was defined as the time-difference in phase between salient features of curves of rainfall and runoff or as the time from center of mass of rainfall to center of mass of runoff. As used in this paper, the term "lag" is defined as the time between center of mass of surface-runoff-producing rain of a specified type of storm and the occurrence of resulting peak-discharge at the location being studied. It is assumed that the lag is constant for a particular drainage-basin. The above definition of lag is consistent with the use of different values of lag on the same area depending on the type of storm.

The lag for any area is obtained either through available records of rainfall and stream-flow or by special field-observation for a few storms. In any case an approximate determination of lag can be made from information readily available.

The factors usually considered as affecting unit-graph shapes are area, shape of basin, topography, channel-slopes, stream-density, and channel-storage. In this study which is based mainly on records of streams located in the Appalachian Highlands, topography, channel-slopes, and stream-density have been eliminated.

Topography was experimentally expressed as different factors, none of which correlated so as to improve the determination of lag. The fact that surface-runoff ceases soon after end of rainfall and is in a channel of some sort within a short time may explain the failure, especially as the size of the area increases.

Channel-slopes likewise failed to be of much help, although provision has been made so that a coefficient can be varied when changing from one general area to another with great differences in channel-slopes.

Channel-storage is generally not obtainable from available maps, although approximate values

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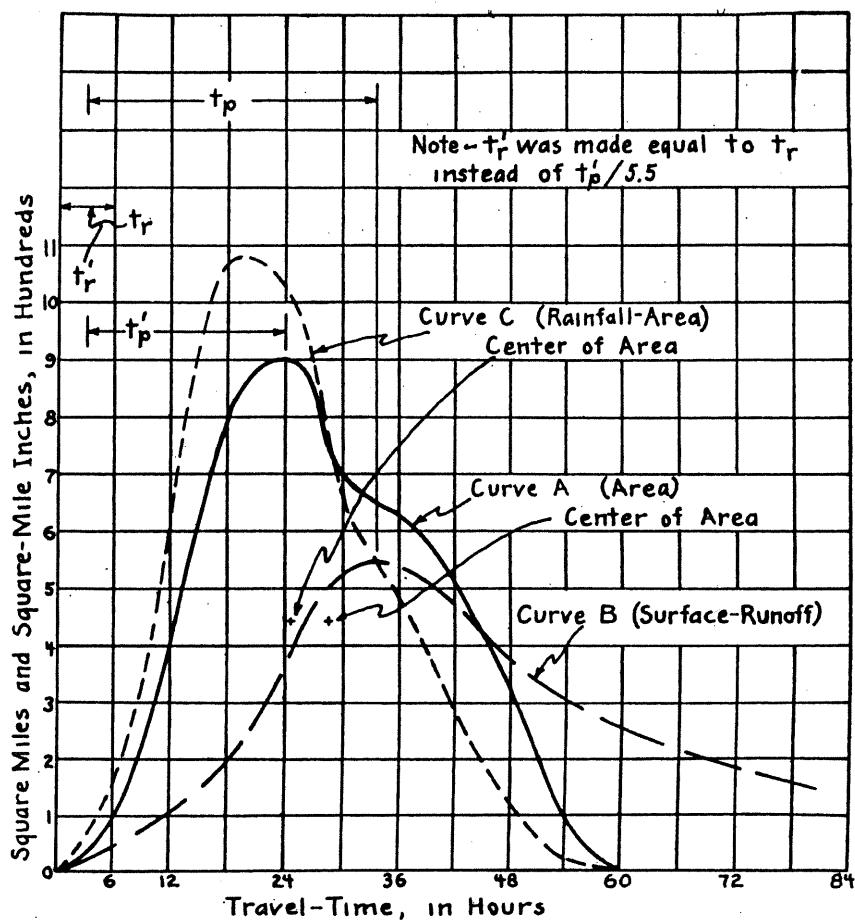


FIG. 1 - SHAPE-DIAGRAMS - FRENCH BROAD RIVER BASIN
ABOVE DANDRIDGE, TENNESSEE

can be derived from stream-flow records. To date a readily obtainable factor to represent the effect of storage on lag has not been developed so its influence was also thrown into the coefficient since channel-slopes and storage are closely related. This leaves size and shape to be considered.

Width of drainage-basin is recognized as one measure of peak-flow to be expected. An additional factor to complete the shape-picture is the distance from the river-measurement station to the center of area. Let us now consider what is meant by width and shape of a drainage-basin.

One way to express the effective shape or width of a drainage-basin is to locate contours of travel-time above the point being studied. These contours can be located by hydraulic computations for steady flow using the Manning or Kutter formula. The results when plotted with time as abscissa and increments of area between time-contours as ordinates express the shape of a drainage-basin by giving the increment of the area that is at any particular travel-time distant from the station. An approximate shape may be obtained by eliminating consideration of shapes and slopes of channel and locating contours of distance from the station and then plotting drainage-area or width (increment of area divided by increment of distance from gage) against distance from the station. None of the above diagrams will show the effect of channel-storage or flood-wave travel.

Curve A of Figure 1 is an area-shape diagram for the French Broad River above Dandridge, Tennessee, based on six-hour travel-time contours and curve B is the six-hour unit-graph for the area. If the unit-graph shape for the area were to be that of curve A, the lag for a six-hour storm would be equal to the time-distance to the maximum ordinate less three hours ($6/2$). However, with the flood-wave action counteracted by the storage-effect that is always present in natural drainage-basins the time-lag will always be greater. In addition, the effects of flood-wave and storage are such that the time-abscissa of the center of area of curve A is a better index of the time of peak-flow than is the time-abscissa of the peak-ordinate of curve A. However, for the example of Figure 1, the time-abscissa of the center of area of curve A is also less than the time of peak-flow of the unit-graph.

The time required for derivation of area-shape diagrams for the basins studied would have been prohibitive, so a substitute for the time-abcissa of the center of area of area-shape diagrams was required. The distance along the principal channel to the actual center of area of the drainage-basin was adopted as the substitute. The center of area for any basin is readily obtained by tracing its outline on a sheet of fairly stiff paper. The basin-map is then cut out and suspended by means of a pin through a point near the edge and a vertical line determined. Two other vertical lines similarly obtained provide an intersection and a check at the center of area.

If the area is irregular with fair-sized tributaries flowing in directions at variance with the general pattern or has two main streams flowing from opposite directions, the center of area must be obtained for the various subareas and a combination made, weighted according to area and distance from the discharge-point.

Where the station is such as the Ohio River at Pittsburgh, formed by the junction of the Allegheny and Monongahela rivers, it is more satisfactory to obtain the lag and also the unit-graphs separately for the two areas. If a single unit-graph is necessary the separate graphs can be combined, although storm-variation can be better taken care of by using the graphs separately.

The following symbols are now introduced:

- t_p = "Lag" in hours
- t_r = Unit of duration of surface-runoff-producing rain in hours (= $t_p/5.5$ in this study)
- t_R = Length of surface-runoff-producing rain in hours
- T = Time-base of unit-graph in days
- L = Length of area in miles
- L_{ca} = Distance from station to center of area in miles
- a_p = Effective area contributing to the peak-flow in square miles
- q_p = Peak-rate of discharge of unit-graph in cfs per square mile
- Q_p = Peak-rate of discharge of unit-graph in cfs
- A = Drainage-area in square miles
- C_t and C_p = coefficients depending on units and drainage-basin characteristics

The addition of an accent (') to the above symbols does not change the definition except to indicate that it refers to a hypothetical runoff-shape as expressed by an area-shape diagram. The unit of duration, t_r , was made a fixed part of t_p to satisfy the requirement of a practical relation between the unit of time used and drainage-basin characteristics.

The use of the expression "surface-runoff-producing rain", in the definitions given above is necessary to eliminate storms or parts of storms that cause practically no surface-runoff.

L_{ca} should be the actual distance on the river from the station to the center of area. If the center of area does not fall on the main channel, the distance should be measured along the main channel to a point opposite the center of area. L should be measured in the same manner until past the center of area after which less and less attention need be paid to minor sinuosities of the stream.

From the study of a large number of basins it was found that t_p varied with $L_{ca}^{0.6}$. The significance of the exponent 0.6 can not be definitely stated at this time as there are a number of possible explanations. Since L_{ca} is an approximation of the actual factor desired the relation is made more stable by introducing L which can be more definitely determined. Then

$$t_p = C_t(L_{ca}L)^{0.3} \quad (1)$$

The value of C_t varied from about 1.8 to 2.2 with an average of 2 for the areas studied which are mainly in the Appalachian Highlands. The coefficient C_t takes care of differences in slopes and storage and did not vary greatly over areas studied, all of which were of somewhat similar characteristics.

The value of t_p thus obtained is the lag for a storm of uniform intensity and distribution. For storms of unequal areal distribution, a rainfall-area-shape diagram can be constructed on the same basis as curve A of Figure 1, but plotting square-mile-inches as ordinates instead of square miles. Curve C, Figure 1, is the rainfall-area-shape diagram for a storm that has two inches of precipitation at the outlet and grades off uniformly to no rainfall on the remote headwater-areas.

Then the time-difference between the center of areas of the rainfall-area-shape and the area-shape diagram is the approximate correction to be applied to the time-lag. A final adjustment would still have to be made for movement of storms up, down, or across the area. This correction consists of an additional adjustment to be applied to the lag, the amount of which would depend mainly on judgment and observed data.

Determination of peak-flow (per square mile) of unit-graph--Practically all hydrographs of surface-runoff from single isolated storms have great similarity. If length of storm, amount of runoff, and size of area are eliminated and the effect of storage and distance to center of area of the basin are represented by the lag, t_p , it is not surprising that a simple relation exists between q_p , t_p , and C_p .

First, taking the hypothetical runoff-case as indicated by an area-shape diagram and since q_p is the peak rate of runoff per square mile per inch of total surface-runoff and since one inch of runoff per hour from one square mile equals 640 cfs, we have $q_p = (640/tr')(a_p/A)$. Then, if C_p equals $(t_p/t_r')(a_p/A)$, $q_p = C_p 640/t_p$ and dropping primes

$$q_p = C_p 640/t_p \quad (2)$$

C_p now equals $5.5 a_p/A$ (in this study t_p/t_r has been fixed at 5.5) but with the flood-wave and storage-transformation of the area-shape diagram into the unit-graph shape, the physical significance of the effective area, a_p , which is causing the peak-runoff of the unit-graph is practically eliminated. For this reason a more satisfactory viewpoint is obtained by introducing a drainage-basin factor, K , such that $K = a_p/a_p = C_p/C_p$. The quantity $(1 - K)$ then expresses the net reduction of the peak-value of the unit-graph by the flood-wave and storage-effect of the drainage-basin.

As developed empirically for the basins studied, the value of q_p varied from $360/t_p$ to $440/t_p$ with an average value of $400/t_p$. This is equivalent to a range in C_p from 0.56 to 0.69. As a rule C_p approached its larger value when C_t approached its lower value. The equation $q_p = C_p 640/t_p$, being an equilateral hyperbola, plots as a 45°-line on logarithmic paper.

Duration of surface-runoff--An expression for the duration of surface-runoff, T , is necessary in constructing Figure 2 which gives values of a distribution-graph for any basin as a function of the lag. This diagram is somewhat similar to that published by Bernard [2] which was based on drainage-basin features. The empirical expression

$$T = [3 + 3 (t_p/24)] \quad (3)$$

was adopted to define a practical value of T . Other values could be used without difficulty and the best duration depends on the method adopted for handling base-flow.

It will be noted that for the smallest area considered (about ten square miles) the value of T will be at least three days. This is the time required for the stream-discharge from a unit-storm to rise and recede to a normal ground-water depletion. Since this is a longer period than can be explained by the effect of channel-storage, it appears that the additional time required must be due to ground-storage or delay. In other words, part of the water which has been included in unit-graphs is not really surface-runoff as usually defined (storm-water flowing over the surface of the ground into open drainage-channels.)

This particular feature has provided difficulties for the author and other investigators for several years. The temporary ground-water flow or better still "subsurface storm-flow" has been defined by C. R. Hursb [5] as "that portion of the storm-flow which infiltrates into the surface soil, but moves away from the area through the upper soil-horizons at a rate much in excess of normal ground-water seepage". It should be added that as pointed out in Water-Supply Paper 772 [3, p. 113], subsurface storm-flow may reach stream-channels with a responsiveness that is only somewhat less pronounced than that which characterizes surface-runoff.

At the present stage of development the most practical procedure is to include most of the subsurface storm-flow in a unit-graph along with the true surface-runoff. This is the result obtained when the ground-water flow is separated from the flood-runoff by means of a normal ground-water depletion-curve. The subsurface storm-flow appears mainly in the recession-side of a unit-graph and necessitates a longer time base than would ordinarily seem necessary. The effect, so far as it influences unit-graph procedure, decreases in importance as size of area increases.

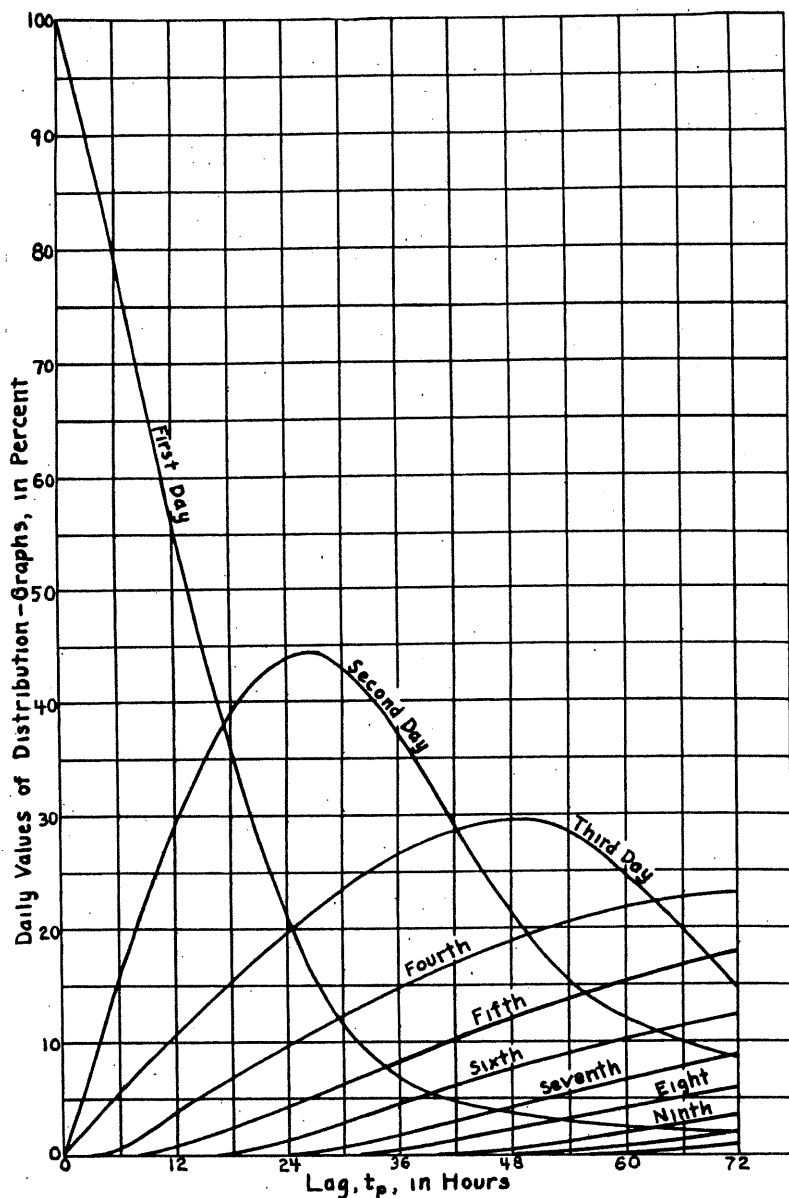


FIG. 2- RELATION OF BASIN-LAG AND DISTRIBUTION-GRAPH

area gives the peak-value of the unit-graph, Q_p , and Figure 2 gives daily values of the distribution-graph percentages.

It should be pointed out here that the distribution-graphs as represented by Figure 2 are averages for typical areas and should be used merely as guides in drawing unit-graphs. If necessary, variations should be made according to the shape-diagram of the area being studied. An example is now given for the French Broad River above Dandridge, Tennessee, whose drainage area is 4,450 square miles.

From a 1/500,000-scale map the length, L , was measured as 130 miles with a map-measurer. The value of L_{ca} was determined as 67 miles with no adjustments since the stream pattern is uniform. Then $t_p = 2 (67 \times 130)^{0.3} = 30.4$ hours; $q_p = 400/30.4 = 13.2$ cfs per inch per square mile; $Q_p = 4450 \times 13.2 = 58,700$ cfs; $t_r = 30.4/5.5 = 5.5$ hours (use nearest whole value, 6); $T = [3 + 3 (30.4/24)] = 6.8$ days

From Figure 2 the distribution-graph percentages for a lag-value of 30.4 are 11.1, 42.6, 23.7, 12.5, 6.4, 3.0, and 0.7. Figure 3 shows the synthetic unit-graph which satisfies the above requirements and whose volume equals 120,000 sec-ft-days (1 inch on 4450 square miles).

Also on Figure 2 is a unit-graph derived from the storm of January 18 to 19, 1936. This storm consisted of four short periods of intense rainfall and lasted 18 hours. The surface-run-

The above procedure will prove satisfactory for most ordinary storms, since it now appears that up to a certain amount of total runoff there is a fairly consistent relation between amounts of surface-runoff, subsurface storm-flow, and ground-water recharge.

Difficulties arise in very intense storms on areas as large as 3,000 square miles. In the same way that it appears probable that most ground-water aquifers have a maximum storage and discharge [6], it is also likely that there is a maximum amount of subsurface storm-water storage and flow. Thus in very large floods due to the maximum being reached or a limit set by existing infiltration-capacity, a greater percentage of the total flood-runoff occurs as true surface-runoff as compared to subsurface storm-flow than is usually the case. This gives a unit-graph with an earlier and higher peak and less volume in the recession-side than is the case in ordinary floods.

In forecasting very large floods having a flood-runoff in excess of a certain amount, the excess or true surface-runoff can be distributed by means of an additional unit-graph.

Example of procedure--There is now available the lag, t_p , which plus $t_r/2$ gives the time from beginning of surface-runoff to the peak of the unit-graph. The rate, q_p , multiplied by the

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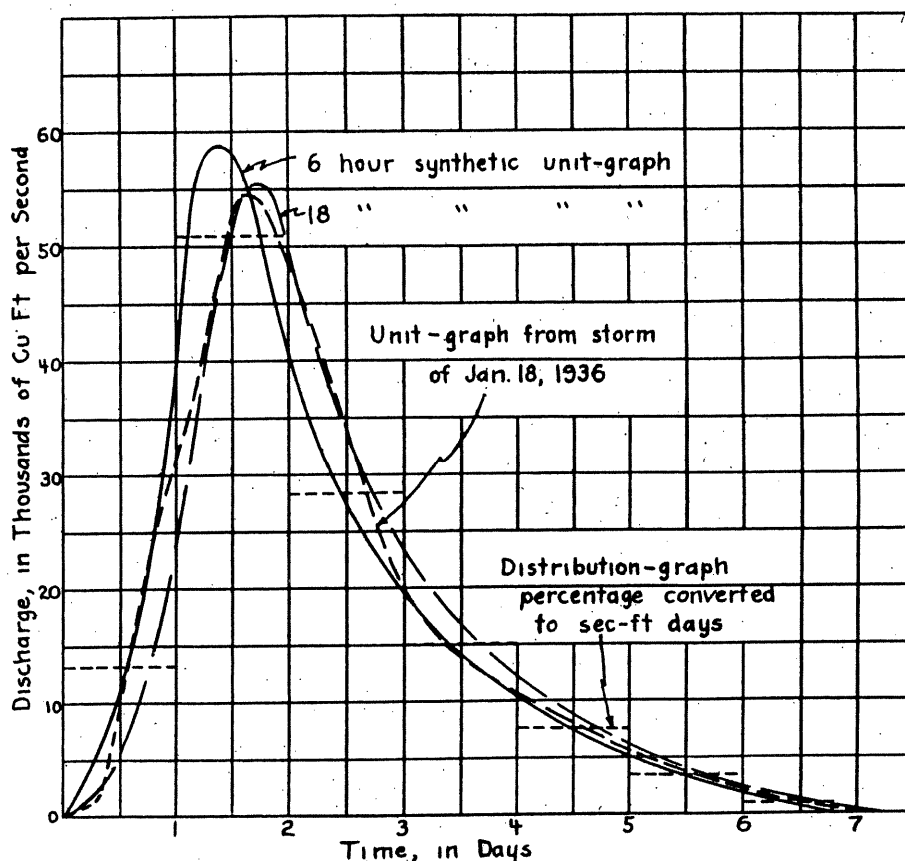


FIG. 3 - SYNTHETIC UNIT-GRAPHS - FRENCH BROAD RIVER BASIN
ABOVE DANDRIDGE, TENNESSEE

off was 1.43 inches. An 18-hour synthetic unit-graph derived from the six-hour synthetic unit-graph is shown for comparison.

Use of procedure in flood-forecasting--In forecasting flood-flows and stages the flow at the point being forecast is usually known up to the time any particular forecast is to be issued, thus outlining the beginning of the rise. If the time of peaking and peak-flow are known it is usually easy to sketch in the intervening rise. Assuming a knowledge of the rainfall and expected amount of total surface-runoff, the time and value of the peak-flow can be closely approximated by knowledge of the expected lag and use of a modified equation (2) which is for one inch of surface-runoff per square mile. By expected lag is meant the lag for a specified type of storm adjusted for actual distribution of rainfall and storm-movement. On any area for which forecasts are being made the value and variation of the lag under different conditions would soon be known from observation even if original values were estimated.

The time-interval, t_r , as used in this paper is about the correct time-interval for collection of data that is necessary for satisfactory forecasting. Actual lengths of storms are not apt to be equal to t_r so equation (2) is modified to form

$$q_{pR} = C_{pR} 640 / [t_p + (t_R - t_r) / 4] \quad (2a)$$

where the variables and coefficients have the same general meaning as before except that the addition of the subscript "R" indicates the length of rain is other than t_r .

Based on the idea of time-contours as expressed by the area-shape curve of Figure 1, expression (2a) would read $q_{pR} = (C_{pR} 640 / t_p)$ where $C_{pR} = 5.5 a_{pR} / A$ and a_{pR} equals the area contributing to the peak-flow and corresponding to the actual storm-duration. However, due to the lack of sufficient area-shape diagrams it was thought best to vary the denominator of the expression rather than the numerator to take care of various duration storms. Equation (2a) is an attempt to do this.

According to the original assumptions, the time of peaking will still be distant from the center of mass of surface-runoff-producing rain by an amount equal to the lag. This assumption differs slightly from the unit-graph procedure. For it to hold in the unit-graph method, the as-

cending and descending slopes of the unit-graph would have to make approximately the same angle with the vertical. This is seldom the case.

To clarify this point, consider a typical area whose four-hour unit-graph peaks in 26 hours from the beginning of rise. The lag is then 24 hours ($26 - 4/2$). Now, the resulting hydrograph obtained by the unit-graph procedure from a uniform 24-hour storm would peak approximately 27 hours later than the center of mass of surface-runoff-producing rainfall. This is three hours later than obtained by the idea of a constant lag of 24 hours. So far it has not been possible to determine which is the more correct as extended uniform storms seldom occur. When two or more separate storms occur close together it is probably more satisfactory to use the regular unit-graph procedure in forecasting.

Limitations and conclusions--(1) It cannot be repeated too often that the unit-graph procedure is a means of determining the time-distribution of surface-runoff, but does not solve the problem of determining how much runoff will occur under any given set of conditions.

To the writer's knowledge, no entirely satisfactory method of determining the amount of surface-runoff from areas of appreciable size has been published. By satisfactory is meant a procedure that would enable one to compute from rainfall day by day the runoff from an area for as many years as desired with a high degree of accuracy and in a reasonable time. It is believed, however, that with the increase in accurate data becoming available for study and with the interest being shown in such studies that the problem will soon be satisfactorily solved and demonstrated.

(2) The lag for a particular drainage-basin is apt to be slightly greater for small floods than for large floods occurring from a given type and duration of storm.

(3) The greater the variation of any particular area from a typical fan-shape, the greater is apt to be the discrepancy between the synthetic and actual unit-graphs.

(4) Extreme care must be used in applying unit-graphs satisfactory for ordinary storms of less than two inches surface-runoff to storms having three or more inches surface-runoff.

(5) The equations and coefficients given are based mainly on the fairly mountainous Appalachian Highlands and may need considerable adjustment to take care of streams in the relatively flat middle west.

(6) It is believed that the procedure presented in this paper will be of material assistance to others as it has been to the author in studies of flood-control, flood-routing [7], and flood-forecasting. The unit-graphs on which the results are based are not presented herein as it is felt that anyone planning to use a unit-graph for any particular area should himself work up the various storms available and become acquainted with the limitations and difficulties of the particular area involved.

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